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Unsteady Airloads in Supercritical Transonic Flows

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Results obtained from a simplified theory of unsteady perturbations of supercritical two-dimensional transonic flows, introduced in an earlier paper, are presented. Unsteady loads generated by an oscillating flap and by airfoils oscillating as a whole are given with comparisons to experimental results and finite-difference solutions. The theory, which was originally formulated for symmetric airfoils at zero mean angle of attack, is extended to treat asymmetric mean flows.

Introduction

THE difficulty in predicting unsteady aerodynamic loads at transonic speeds lies in the controlling effect which small local nonuniformities in the steady flow have on the behavior of receding acoustic waves. This effect is especially important in supercritical ("subsonic transonic") flows since embedded supersonic regions will reflect and diffract (but will not transmit) any radiation incident from downstream. Such supersonic bubbles are normally terminated by shock waves which move about in response to the acoustic field (in accordance with the requirements of the conservation laws), thereby inducing large amplitude loads over small regions of the surface. The appearance of shocks in the steady flow is largely responsible for the often abrupt failure of classical unsteady small-disturbance theory at the critical Mach number.

In an earlier paper,[1] a simple model of two-dimensional supercritical flows with embedded shocks was formulated from which unsteady loads on harmonically oscillating airfoils may be readily calculated. The model employs the standard approximations of transonic small-disturbance theory (e.g., mean surface approximation, linearized Bernoulli equation, etc.) together with the following additional assumptions and approximations:

1) Infinitesimal unsteady amplitudes. That is, we deal with linear acoustics in a nonuniform, possibly discontinuous, mean flow as discussed in Ref. 2.

2) Symmetric mean flow. The undisturbed airfoil is presumed to be symmetric and at zero angle of attack.

3) Shocks are infinite in length and normal to the freestream.

4) The effects of continuous mean flow nonuniformities can be modeled by Dowell's method[3] of "local linearization."

In consequence of assumption 3 (which is equivalent to neglecting diffraction about the shock tip) the acoustic field upstream from the shock is completely independent of that downstream and is given, essentially, by classical supersonic linear theory. Application of the conservation of mass across the shock, then, provides a known source-like boundary condition along the shock for the elliptic boundary-value problem downstream. The solution in this region is constructed by placing a vortex sheet of unknown strength along the airfoil and wake (as in classical subsonic theory) and a known source distribution along the shock. The sources induce an additional upwash on the airfoil which must be removed by adjusting the vortex sheet strength so that the combined field satisfies the prescribed normal velocity boundary condition on the airfoil. Finally, the shock motion is determined from the conservation of transverse momentum, once the acoustic fields upstream and downstream of the shock have been evaluated.

It is presumed that the steady-state pressure distribution on the airfoil (most importantly, the steady shock strength and position) are known in advance. This information may be taken from any available source, theoretical or experimental. Use of experimental input data provides an automatic "correction" for boundary-layer and wind-tunnel wall effects, which are difficult to predict theoretically but can have a significant influence, especially on shock position.

Specific results based on this model were given in Ref. 1 for the case of a fixed airfoil with oscillating quarter chord flap, the configuration studied experimentally by Tijdeman[4,5] and numerically by various authors.[6-9] All the results given by Ref. 1 were for freestream Mach numbers at which the shock stands upstream from the flap hinge, so that, within the context of the model, the flow upstream from the shock is disturbance-free (by assumption 3 above).

In the present paper additional results will be given for the oscillating flap problem when the shock stands downstream of the hinge and for airfoils oscillating as a whole. In addition, the application of the model to asymmetric mean flows (cambered airfoils and/or nonzero mean angle of attack) will be considered. Finally, the limitations of the model, particularly with regard to the presumed absence of upstream influence around the shock, will be critically discussed.

Results

The experimental results obtained by Tijdeman and Schippers,[4] for the 64A006 airfoil with quarter-chord flap indicate that at zero flap deflection the shock stands on the flap for freestream Mach numbers between 0.92 and 1. Within this Mach number range the forward portion of the control surface is exposed to locally supersonic conditions, the aft portion to locally subsonic conditions. For this situation the theory of Ref. 1 predicts that when the flap is oscillated there will be no disturbances upstream from the hinge axis and that the load induced between the hinge and the shock will be given (essentially) by classical supersonic theory, while the load between the shock and trailing edge will be given (essentially) by classical subsonic linear theory modified by wave reflection and refraction by the shock.

We consider first the case $M_\infty = 1$, for which the shock stands sensibly at the trailing edge and the entire flap is exposed to supersonic flow, as indicated by the steady pressure distribution shown in Fig. 1. For a quasisteady flap deflection δ (the only result reported by Tijdeman at this Mach number) the theory predicts the simple wave load $\Delta Cp = 4\delta/\beta$, where $\beta \equiv (M_{TE}^2 - 1)^{1/2} \doteq 0.75$. The agreement between this value and the measured load is shown in Fig. 2. It seems likely that good agreement would also be obtained at nonzero frequencies.

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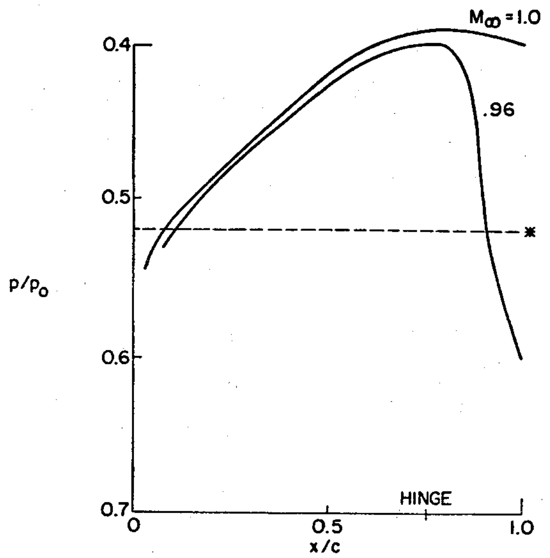


Fig. 1 Steady pressure distribution on NACA 64A006 near Mach 1 for $\alpha = \delta = 0$ (from Ref. 4).

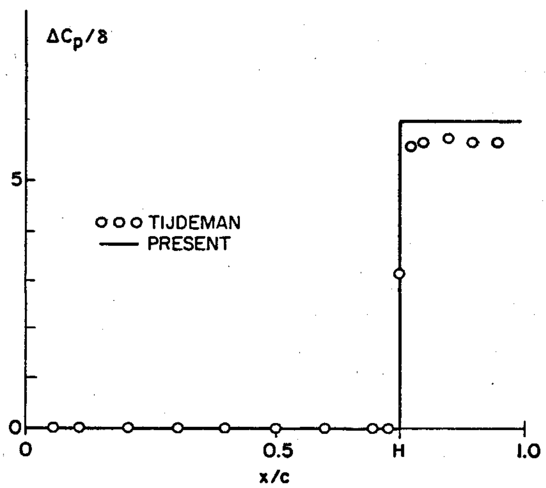
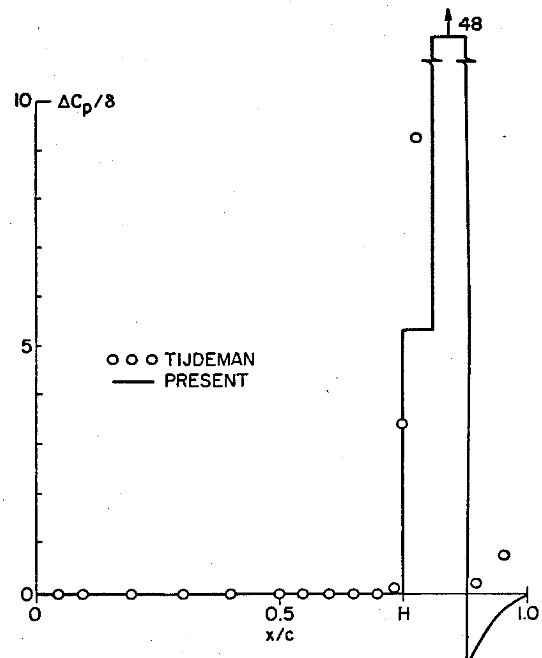
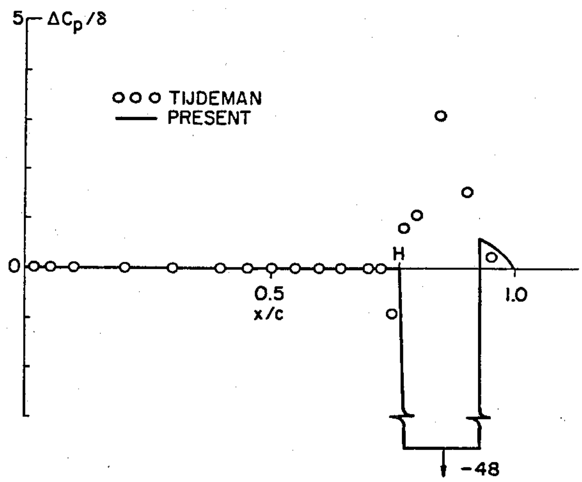


Fig. 2 Quasisteady load due to flap deflection at $M_\infty = 1$, $\alpha = 0$, $\delta = 1$ deg, $k = 0$.



a) Real part



b) Imaginary part

Fig. 3 Unsteady load due to flap oscillation of $M_\infty = 0.96$, $\alpha = 0$, $\delta = 1$ deg, $k = 0.05$.

It should be noted that for this case the normal shock approximation used in the model is probably very inaccurate. However, at $M_\infty = 1$ disturbances in fact cannot propagate upstream, so that the actual shock geometry cannot influence the load on the airfoil. Moreover, the disturbed region near the flap is embedded well within the supersonic region so that the quasiuniform mean flow approximation used in the model is justified locally. Hence the solution obtained is accurate in the vicinity of the airfoil, even though it would be poor in the far field downstream.

The situation at freestream Mach numbers slightly below 1 is considerably different. In this case the undisturbed shock stands forward of the trailing edge, *separating* the boundary layer downstream. The shock/boundary-layer interaction is spread out over most of the flap chord, as indicated by the measured surface pressure distribution for $M_\infty = 0.96$ shown in Fig. 1. Since the flow over the flap is dominated by the shock/boundary-layer interaction, it is unlikely that *any* purely inviscid theory can successfully predict the unsteady loads. The prediction of the present theory for an oscillation at 30 Hz is shown in Fig. 3. For comparisons with experiment[4] and other theory[8,9] at lower Mach numbers, see Ref. 1. Note that the theory of Ref. 9 is a simpler version of that of Ref. 8.

We note that the theory predicts a large peak associated with the shock's motion, a feature which is characteristic of inviscid theories and of measurements at lower supercritical Mach numbers, but that the data for this case show no evidence of such a peak. These pressure spikes are most pronounced when the shocks are true discontinuities, since a given point will then "see" an instantaneous jump in pressure as the shock passes over. If the shock is smeared out by a boundary layer (or, for that matter, by a coarse finite-difference grid), the pressure excursions at the point become smaller and the spike tends to disappear. The height of the peak depends strongly on the ratio of the shock thickness (based on maximum pressure gradient, for example) to the amplitude of its motion. If the ratio is large (broad shocks or small displacements), the peak disappears; if it is small (thin shocks or large displacements), the peak pressure will be nearly equal to the equilibrium pressure jump across the shock.

Aside from this, the assumptions of the model are reasonably well met: the flap actually is surrounded upstream by a large supersonic region into which disturbances cannot propagate. Hence both theory and experiment show no influence ahead of the hinge. It is likely that major improvements in the predicted loads on the flap would be achieved if a *boundary-layer model* were incorporated into the theory.

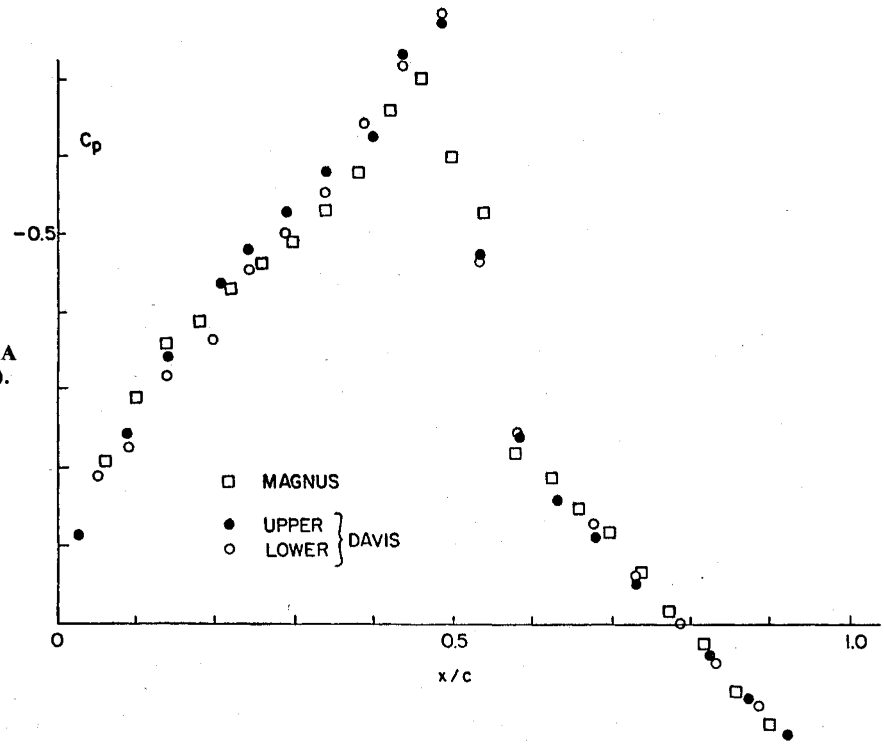


Fig. 4 Steady pressure distribution of NACA 64A010 at $M_\infty = 0.8$, $\alpha = 0$ deg (from Refs. 6 and 8).

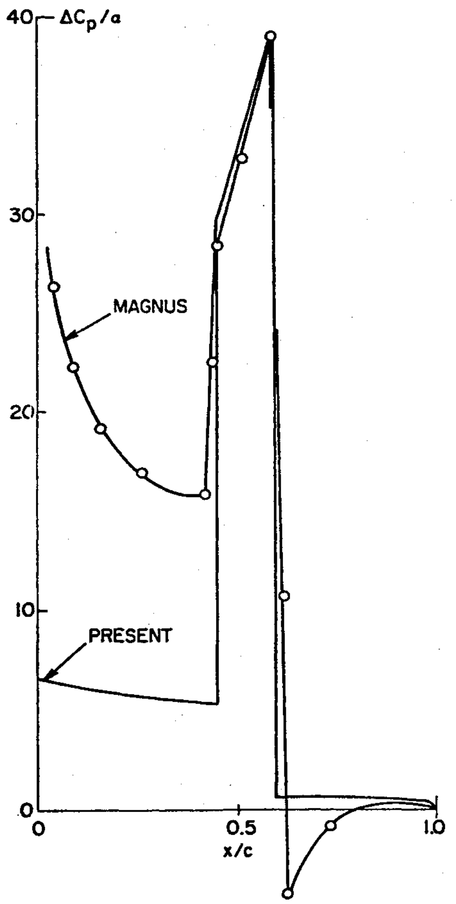


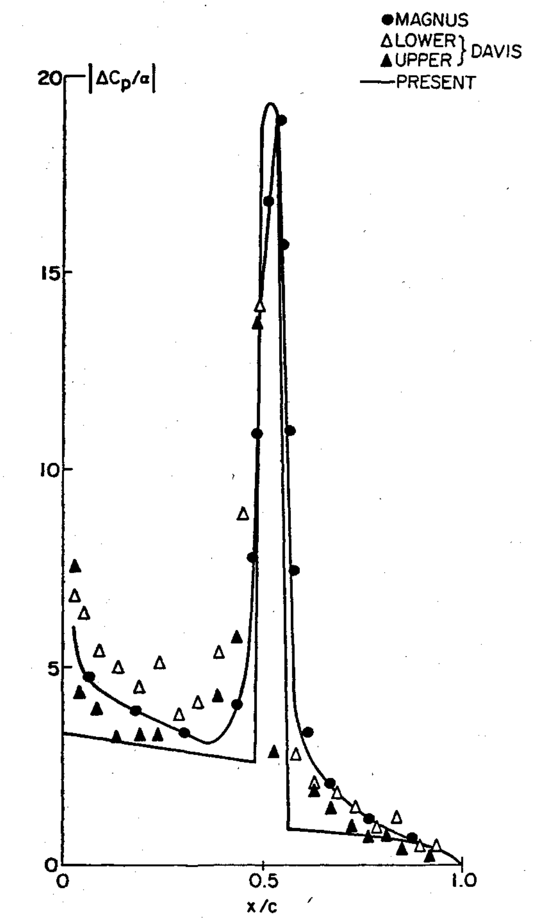
Fig. 5 Quasisteady load for $\alpha = 1$ deg about mean flow of Fig. 4.

The model treats the entire region upstream of the shock as supersonic. Thus the *mathematical* problem for an airfoil oscillating as a whole is precisely the same as that for the oscillating flap with the shock downstream from the hinge. Of course, the physical problems are different: the flap really does have a supersonic leading edge, while the airfoil's is obviously subsonic. Despite this conceptual difficulty the model still seems to work reasonably well for airfoils oscillating as a whole. We will consider here results for the NACA 64010 airfoil at $M_\infty = 0.8$, which has been studied experimentally by Davis[10] and numerically by Magnus.[7]

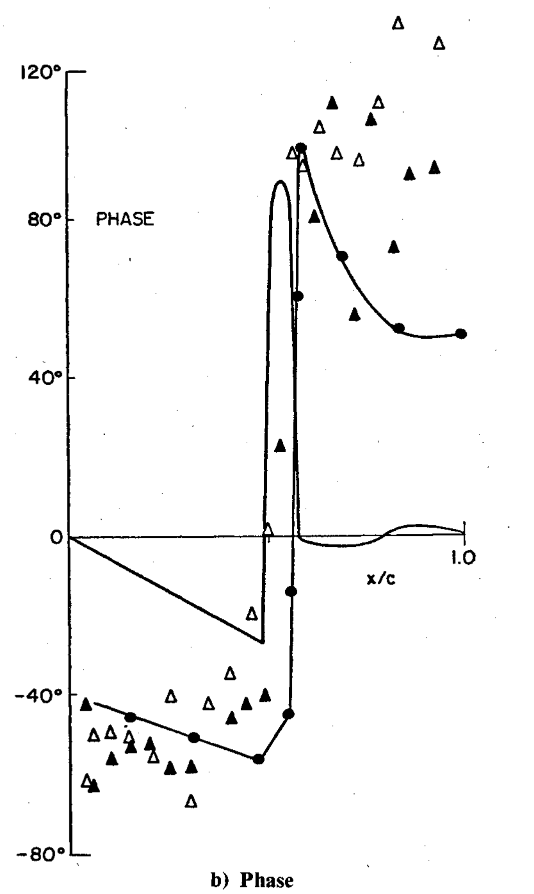
The steady pressure distribution (zero angle of attack) is shown in Fig. 4. The very close agreement between the theory of Magnus and experiment reflects the absence of significant wall interference or boundary-layer effects in the experiment. Note that the present theory uses the data of Fig. 4 as an input.

The load induced by a small quasisteady angle of attack is shown in Fig. 5, together with Magnus' finite-difference solution of the Euler equations. The present model agrees very well with the numerical results for the load induced by the shock motion. The rapid expansion seen just downstream from the shock in the finite-difference solution is caused by the interaction of the shock with the local surface curvature[2] (the "Zierep cusp"), an effect which is not included in the model and which is not observed experimentally because of the boundary layer. Of greater interest is the large discrepancy in the load upstream from the shock, the present result indicating much too small a value. Discrepancies of this sort were also observed in the oscillating flap problem[1] when the shock is upstream from the hinge. This is not a nonlinear effect, since finite-difference solutions of the linearized equations, such as those obtained by Traci et al.,[6] also show larger loads in the supersonic region than would be anticipated from simple wave theory (their predictions also tend to be larger than those observed experimentally). Nor can the discrepancy be removed by using a more refined mean flow description in the supersonic region, accounting for the subsonic leading edge, but still neglecting upstream influence from the trailing-edge region. A model of this sort, with a mean flow continuously accelerating from subsonic to supersonic speeds, was tried but succeeded only in altering the load distribution (introducing a characteristic subsonic leading-edge square root singularity) without changing its overall level.

In the author's opinion this discrepancy is caused by the neglect of upstream influence around the shock tip when the



a) Amplitude



b) Phase

Fig. 6 Unsteady load for $\alpha = 1$ deg, $k = 0.02$ about mean of Fig. 4.

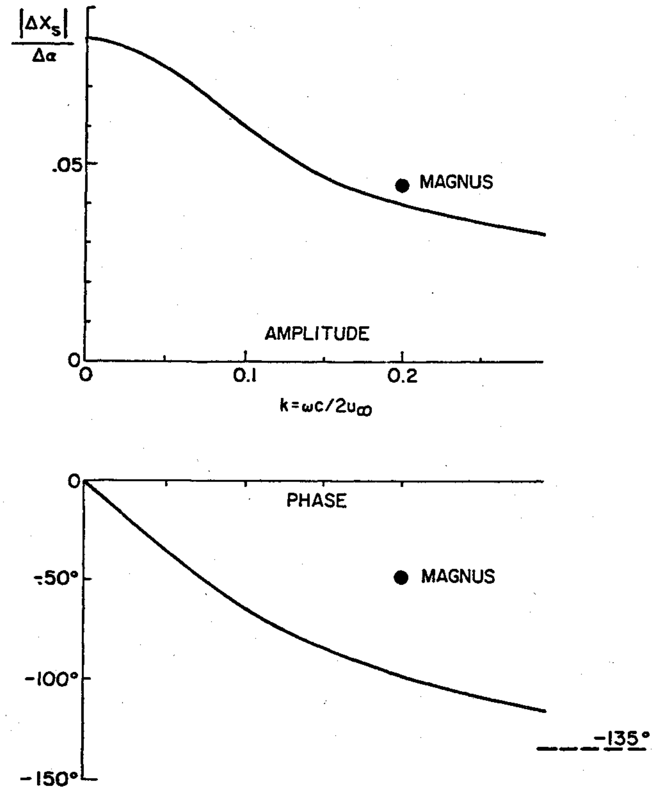


Fig. 7 Amplitude and phase of shock motion vs frequency, about mean of Fig. 4 (present model compared to Magnus).

Kutta condition is imposed at the trailing edge. The "Kutta waves" emitted from the trailing edge by the shedding of vorticity when the angle of attack is changed apparently have a finite (first-order) effect on the load upstream of the shock as the steady limit is approached. This view is supported by the fact that the unsteady solution on the subsonic side of the infinite shock, found in Ref. 1, develops a finite flow velocity (proportional to the lift) along the shock at infinity *as the frequency goes to zero*. Such a flow must come from the shock tip, and presumably induces a corresponding flow out of the supersonic bubble through the sonic line. This in turn would generate disturbances which are carried by the left-running family of characteristics to the airfoil surface in the supersonic region, thereby augmenting the load upstream of the shock. It should be noted that in a nonlifting problem (e.g., a pulsating airfoil) this problem does not arise (the perturbation velocities at infinity are zero in the steady limit, even with an infinite shock) so that upstream influence around a large but finite shock is probably negligible in the absence of lift.

At *finite* oscillation frequencies upstream propagating waves tend to cancel, so that their neglect in the simplified model is more justifiable. This is illustrated in Fig. 6, which shows a comparison of the present model with experimental and finite difference results for the 64A010 airfoil oscillating at a reduced frequency of 0.2. Strictly, the finite-difference solution is for a plunging motion, the experiment for pitching about the quarter chord (the plunge amplitude is such as to give the same equivalent angle of attack as in the experiment at this frequency). The present theory employs a low-frequency approximation and therefore does not distinguish between the two modes, aside from proportionality factors. The approximation is clearly supported by the similarity between the finite-difference and measured loads. The results for $k = 0.2$ are much improved over those for $k = 0$ and, in general, the present model should be better as k increases. Figures 7-9 show predicted shock motion, lift, and pitching moment as a function of reduced frequency. Using

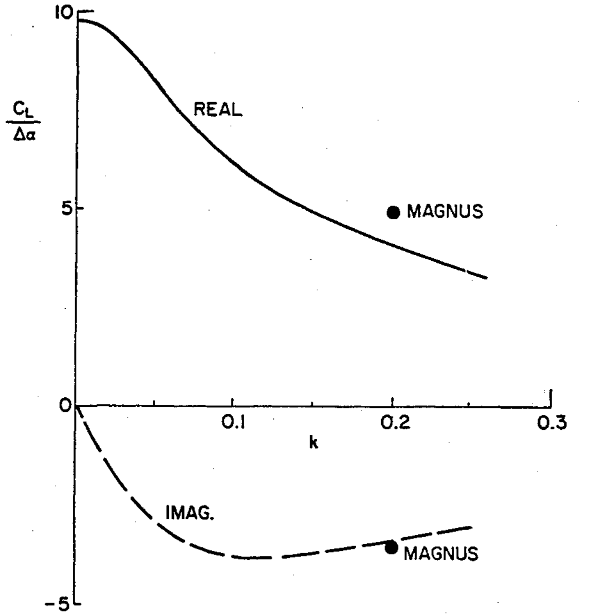


Fig. 8 Real and imaginary part of unsteady lift vs frequency, for mean flow of Fig. 4 (present model compared to Magnus).

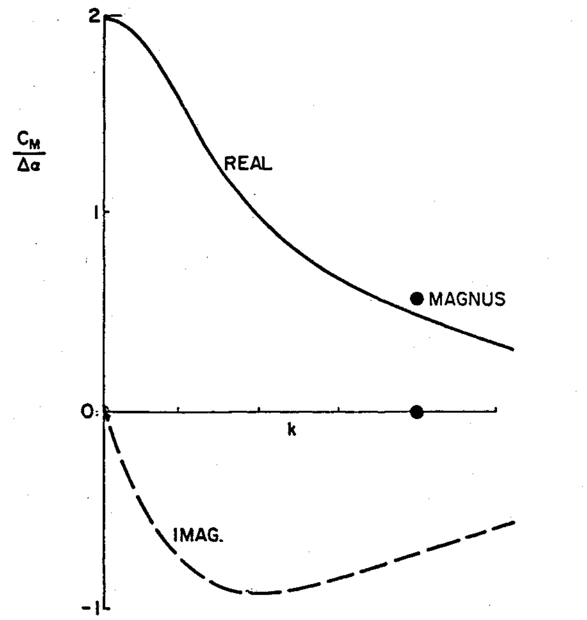


Fig. 9 Real and imaginary part of unsteady pitching moment vs frequency, for mean flow of Fig. 4 (present model compared to Magnus).

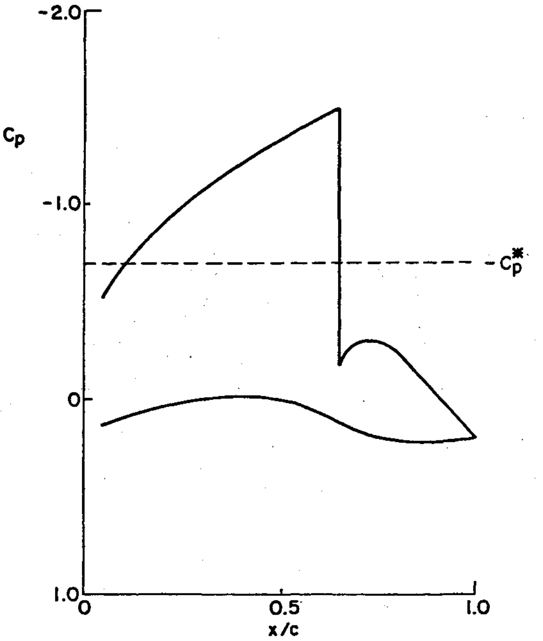


Fig. 10 Steady pressure distribution on NACA 64A410 at $M_\infty = 0.72$, $\bar{\alpha} = 1$ deg (from Ref. 6).

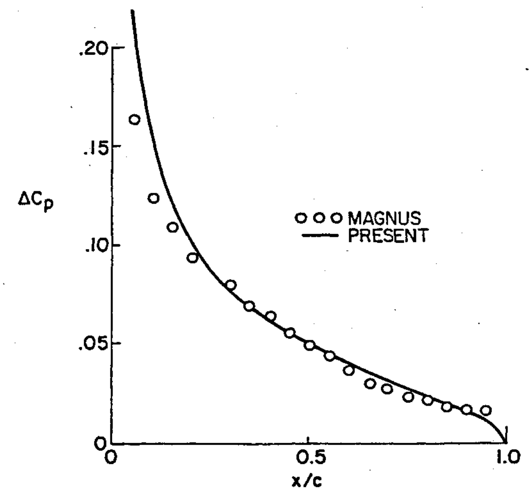


Fig. 11 Quasisteady load on lower surface, for mean flow of Fig. 10.

Magnus as the standard, the present results are best for lift and somewhat less accurate for shock motion and moment. See especially the imaginary parts.

Asymmetric Mean Flows

If the mean flow is symmetric, the linearized problem can be formulated in the half plane with homogeneous boundary conditions (on pressure) on the wake and upstream dividing streamline. If the mean flow is asymmetric, this is no longer possible in general and the upper and lower flows must be evaluated simultaneously, with continuity conditions applied across their common boundaries. *In transonic flows,* however, the communication between the upper and lower surfaces may well be weak, and one may ask whether useful results could be obtained by neglecting the interaction entirely. In practice this means using the solution for each surface determined as if the flow were symmetric, i.e., with homogeneous boundary conditions on the dividing streamline (this will yield discontinuous normal velocities on the dividing streamline but may have little effect on the airfoil surface).

We consider, for example, the 64A410 airfoil at 1 deg mean angle of attack, $M_\infty = 0.72$. The steady-state pressure distribution for this configuration, according to Magnus' finite difference calculations,[7] is shown in Fig. 10.

It will be noted that the lower surface is competely subcritical, with local Mach numbers not far removed from the freestream. The "model" for such a flow is simply classical subsonic linear theory, neglecting the interaction with the upper surface. The corresponding load distributions, for reduced frequencies $k = 0$, 0.2, are shown in Figs. 11 and 12. The agreement with the finite-difference solution is remarkably good, considering that the upper surface is massively supercritical. Similar results have been observed by Tijdeman[5] for for the subcritical lower surface of an NLR7301 supercritical section.

The upper surface of the 410 has a strong shock standing at 0.55 *c*. The results of the simplified model for the load induced by a small change in angle of attack at $k = 0$ are shown in Fig. 13, together with Magnus' solution of the Euler equations and Traci et al.'s[6] finite-difference solution of the

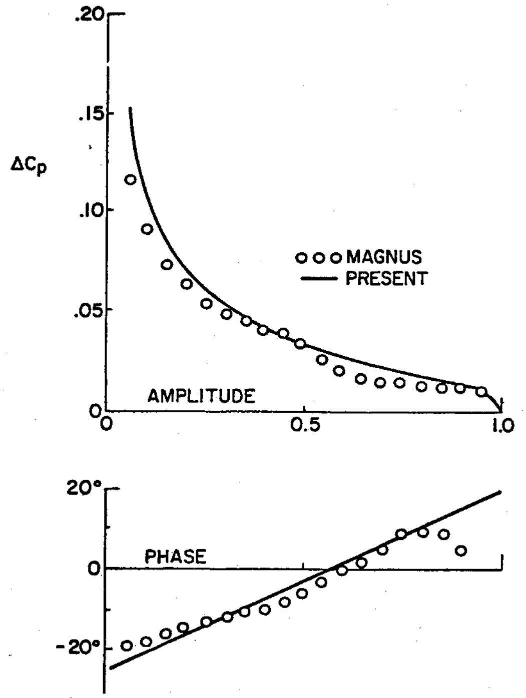


Fig. 12 Unsteady loads on lower surface, $k = 0.2$, for mean flow of Fig. 10.

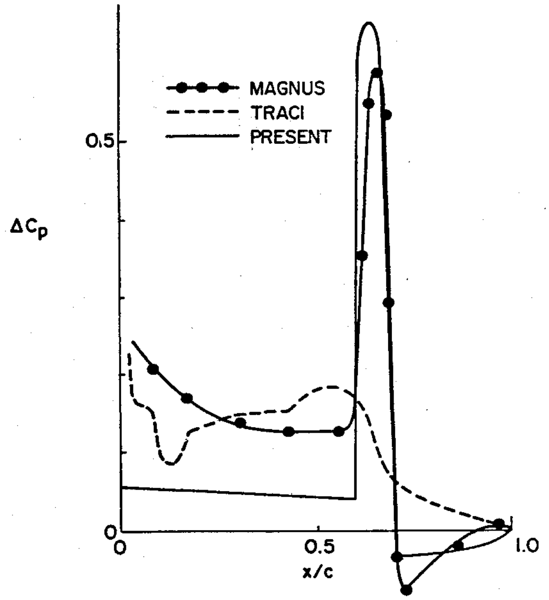


Fig. 13 Unsteady loads on upper surface, $k = 0.2$, for mean flow of Fig. 10.

linearized small-disturbance equation. The accuracy of the model is certainly no worse in this case than for the symmetric airfoil (Fig. 4), the discrepancies being of a similar nature and undoubtedly arising from a similar source (influence around the shock tip). By way of comparison, Traci's solution suffers from the "wrong" mean motion and an inaccurate treatment of the shock by comparison to Ref. 9, for example. However, their solution does predict the right load level within the supersonic bubble.

Conclusions

The simplified model of Ref. 1 has been applied to three problems of general interest: 1) oscillating control surface with shock on flap, 2) oscillating symmetric airfoil, and 3) oscillating cambered airfoil. For the flap case, the theory does well when the shock is at the trailing edge, but fails when the shock is slightly upstream, probably because the flow is dominated by the shock/boundary-layer interaction and *separation*. However, for the shock further upstream the theory again does well. For the oscillating airfoil problems, the model appears to predict the shock motion and its attendant load. The major improvement needed is the inclusion of upstream influence around the shock tip, especially at very low frequencies. Camber and mean angle-of-attack effects can be handled satisfactorily by analyzing the upper and lower surfaces independently. In general, the model is less successful in predicting phase than magnitude of the aerodynamic forces. Also it is more accurate as the reduced frequency is *increased.*

Acknowledgment

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